Name\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Student number\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Assignment 3**

Consider the equations of motion



for the end point rotations of a certain torsion bar of length *L*. Above,  is the second moment of area with respect to the axis of the bar (polar moment), *G* is the shear modulus, and *ρ* is the density of material. Derive the angular speeds and the corresponding modes of the free vibrations.

**Solution template**

The set of ordinary differential equations as given by the principle of virtual work  consists of the inertia and stiffness parts. The symmetric mass matrix  and the stiffness matrix  depend on the structure. Angular speeds of the free vibrations are the eigenvalues of . In practice, it is easier to calculate first the eigenvalues  as the eigenvalues of  are the square roots of those for  and the eigenvectors coincide.

In the present case, the matrices are

 and .

Therefore

.

In the eigenvalue problem of matrix , the goal is to find all pairs  such that . The linear homogeneous equation system can have a non-zero solution only if . The eigenvalues are obtained as solutions to this characteristic equation. The characteristic equation for the eigenvalues of  is

.

The two solutions for the eigenvalues are (  )

 and .

The corresponding eigenvectors are obtained as solutions to . The eigenvectors are not unique and it is enough to find some of them. However, the eigenvectors should be linearly independent so that, e.g., the zero vector is not a valid choice.

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The representation of the matrix in terms of its eigenvalues and eigenvectors  implies that . As taking a square root of the diagonal matrix means just taking the square roots of the diagonal terms, the angular speeds of the free vibrations

 and . 🡸